

Frustration from Simultaneous Updating in Sznajd Consensus Model

Dietrich Stauffer

Institute for Theoretical Physics, Cologne University
D-50923 Köln, Euroland

In the Sznajd model of 2000, a pair of neighbouring agents on a square lattice convinces its six neighbours of the pair opinion iff the two agents of the pair share the same opinion. Now we replace the usual random sequential updating rule by simultaneous updating and find that this change makes a complete consensus much more difficult. The larger the lattice is, the higher must be the initial majority for one of the two competing opinions to become the consensus.

Key Words: opinion dynamics, computer simulation, sociophysics

The application of cellular automata, Ising models and other tools of (computational or statistical) physics has a long tradition (Majorana 1942, Schelling 1971, Sakoda 1971, Callen and Shapero 1974, Galam et al 1981, Schweitzer 1997, Weidlich 2000). In particular one would like to know the conditions to reach a consensus out of an initially diverging set of opinions (Deffuant et al 2000, Kobayashi 2001, Hegselmann and Krause 2002). Most models assume that every agent is influenced by its neighbours and takes, for example, the opinion of the majority of them, or of a weighted average. The Sznajd model (Sznajd-Weron and Sznajd 2000; for a review see Stauffer 2002), on the other hand, assumes that every agent tries to influence its neighbours, without caring much about what they think first. Thus in the Sznajd model the information flows outward to the neighbourhood, as in infection or rumour spreading (Noymer 2001), while in most other models the information flows inward from the neighbourhood. Also, the Sznajd model takes into account the well-known psychological and political fact that “united we stand, divided we fall”; only groups of people having the same opinion, not divided groups, can influence their neighbours.

On the square lattice, where every site is occupied by an agent having one of two possible opinions $+1$ and -1 , the most-studied Sznajd rule is: **A pair of nearest neighbours convinces its six nearest neighbours of the pair opinion if and only if both members of the pair have the same opinion; otherwise the pair and its neighbours do not change opinion.** Initially the opinions are distributed randomly, $+1$ with probability p and -1 with probability $1 - p$. This standard model then gave always a consensus, which for large lattices was that opinion which initially had a majority; if $p = 1/2$ initially, then half of the cases ended with everybody having opinion $+1$, and the other half of the cases with the opposite opinion.

In these simulations random sequential updating was used, i.e. one of the $L \times L$ agents in the square lattice was selected randomly, and then one of its four neighbours to check if they

share the same opinion. One time step was completed if on average each of the $L \times L$ agents was selected once as the first member of the pair. A. Iosselevitch (private communication) suggested to compare this rule with the simultaneous updating traditional for cellular automata, used also in the consensus model of Kobayashi (2001): Each pair is judged by its opinion at time t to gives its six neighbours their possibly new opinions at time $t + 1$. Now we can go through the lattice like a typewriter to find the first member of the pair; only for the second member of the pair a random selection is still needed. Going through the whole lattice once constitutes one time step.

In both random sequential as well as simultaneous updating, an agent can belong to the neighbourhoods of several convincing pairs. For random sequential updating, the agent then follows each pair in the order in which it receives orders, just like civil servants followed their various governments in Germany during the 20th century. For simultaneous updating, on the other hand, it does not know what to do if one pair has opinion $+1$ and another also neighbouring pair has the opposite opinion. It then feels frustrated and does nothing. i.e. it stays with its old opinion. (Similar frustration effects are known from some models of magnetism.) This frustration then hinders the development of a consensus.

With up to 800 samples, and $L \geq 13$ we never found a consensus at $p = 1/2$. One has to use small lattices, or p different from $1/2$, to find all agents at the end having the same opinion. Fig.1 shows how the number (among 800) of samples without a consensus even after 10,000 time steps varies with lattice size L and initial concentration p ; if there was a consensus it was in favour of the initial majority. (For $7 \leq L < 13$ also at $p = 1/2$ rare cases of a consensus were found.) Fig.2 shows how the p needed to get a consensus in half the cases varied for varying lattice size; the problem is by definition symmetric about $p = 1/2$, and only $p < 1/2$ is thus plotted in our figures.

Of course, reality differs from a square lattice. If everybody interacts with everybody else equally, without regard of a geometrical distance (Kobayashi 2001), this “mean field” problem may be suited for analytical solution or description by differential equation. More realistic may be a network where a few agents have lots of neighbours, and most have only a few neighbours, without a sharp boundary between celebrities and common folks (Albert and Barabasi 2002). Here an Ising model gave already an unusual phase transition (Aleksiejuk et al 2002), and a different Sznajd model with many possible opinions agreed with Brazilian election results.

Our simultaneous updating corresponds to formal committee meetings at times fixed for all participants, while random sequential updating corresponds to informal meetings of subgroups at various times. Our simulations then indicate that informal meetings have a higher chance to lead to a consensus.

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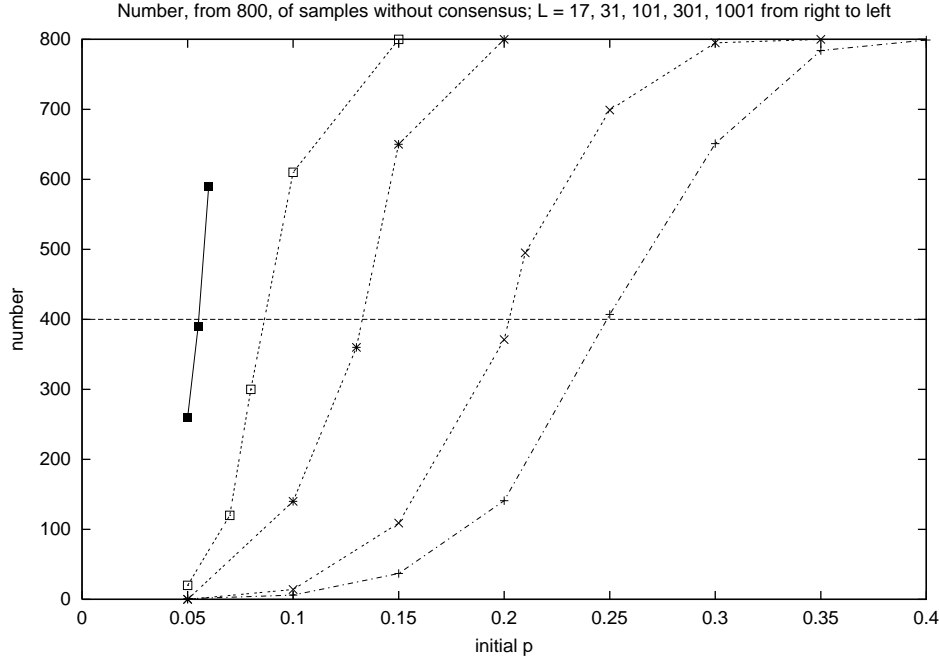


Figure 1: Number, from 800 samples, of cases where still a consensus was reached, for $L = 17$ and 31. For $L = 101, 301$, and 1001 only 80 samples were run and the resulting numbers thus multiplied by 10.

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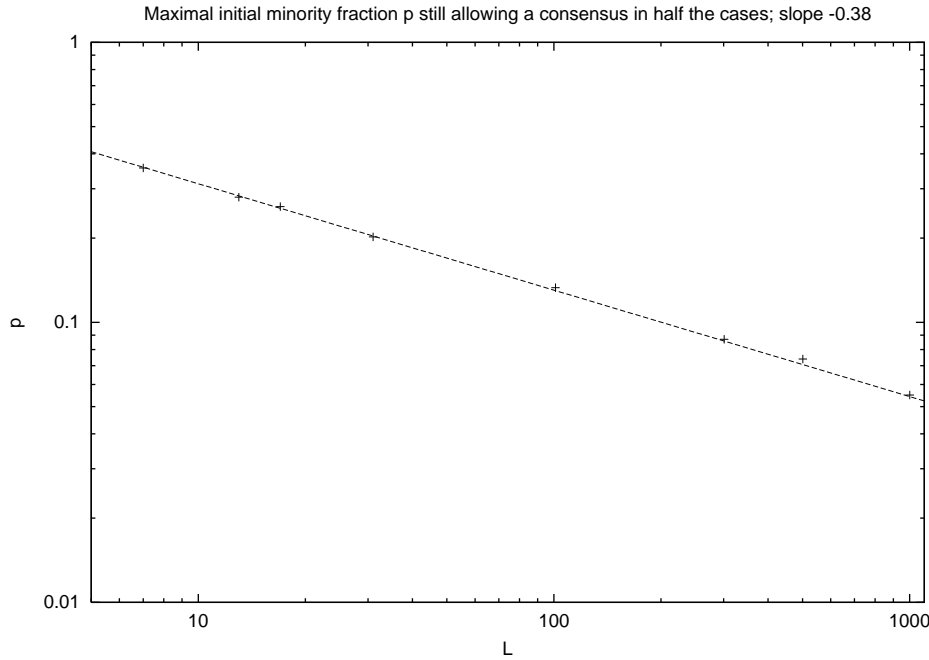


Figure 2: Variation with L of the initial probability for which in half the cases a consensus was reached.

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